Error correction for a proposed quantum annealing architecture

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29 October 2015

Abstract

Recently, Lechner, Hauke and Zoller [1] have proposed a quantum annealing architecture, in which a classical spin glass with all-to-all connectivity is simulated by a spin glass with geometrically local interactions. We interpret this architecture as a classical error-correcting code, which is highly robust against weakly correlated bit-flip noise.

Quantum annealing [2] is a method for solving combinatorial optimization problems by using quantum adiabatic evolution to find the ground state of a classical spin glass. Hoping to extend the reach of quantum annealing in practical devices, Lechner *et al.* [1] have proposed a scheme, using only geometrically local interactions, for simulating a classical spin system with all-to-all pairwise connectivity. Their scheme may be viewed as a classical low-density parity-check code (LDPC code) [3]; here we point out that the error-correcting power of this LDPC code makes the scheme highly robust against weakly correlated bit-flip noise.

Lechner *et al.* propose representing N logical bits $\vec{b} = \{b_i, i = 1, 2, \ldots, N\}$ using $K = \binom{N}{2}$ physical bits $\vec{g} = \{g_{ij}, 1 \leq i < j \leq N\}$, where g_{ij} encodes $b_i \oplus b_j$ and $\oplus$ denotes addition modulo 2. The K physical variables obey $K - N + 1$ independent linear constraints. Hence only $N-1$ physical variables are logically independent; we may, for example, choose the independent variables to be $\{g_{12}, g_{23}, g_{34}, \ldots, g_{N-1,N}\}$. The linear constants may be chosen to be weight-3 parity checks. If weight-4 constraints are also allowed then the parity checks can be chosen to be geometrically local in a two-dimensional array. Higher-dimensional versions of the scheme may also be constructed [1]; we will discuss only the two-dimensional coding scheme here, but the same ideas also apply in higher dimensions.

While g_{ij} denotes the value of $b_i \oplus b_j$ in the ideal ground state of the classical spin glass, we use g'_{ij} to denote the (possibly noisy) readout of the corresponding physical variable after a run of the quantum annealing algorithm. If the readout is not too noisy, we can exploit the redundancy of the LDPC code to recover the ideal value of $\{b_i \oplus b_j\}$ from the noisy readout $\vec{g}'$ with high success probability. Given an error model, we can determine the conditional probability $p(\vec{g}'|\vec{b})$ of observing $\vec{g}'$ given $\vec{b}$. Assuming that each $\vec{b}$ has the same *a priori* probability, we decode $\vec{g}'$ by finding the most likely $\vec{b}$:

$$\vec{b}_{\text{decoded}} = \text{MLE}(\vec{g}') = \text{ArgMax}_{\vec{b}}\ p(\vec{g}'|\vec{b}), \tag{1}$$

where MLE means "maximum likelihood estimate." In fact, we can only recover the ideal $\vec{b}$ up to an overall global flip since one bit of information is already lost during encoding.

We adopt the simplifying assumption of independent and identically distributed (i.i.d.) noise: g'_{ij} is flipped from its ideal value g_{ij} with probability $\varepsilon \leq 1/2$, and agrees with its ideal value with probability $1-\varepsilon$. Though we do not necessarily expect this simple noise model to faithfully describe the errors arising from imperfect quantum annealing, our assumption follows the presentation of [1]. This model might be appropriate if, for example, the noise is dominated by measurement errors in the readout of the final state. It also allows us to estimate $p(\vec{g}'|\vec{b})$, either analytically or numerically. Exact MLE decoding is possible in principle, but has a very high computational cost. We will settle instead for decoding methods which are feasible though not optimal.

There is a very simple error correction procedure for which we can easily estimate the probability of a decoding error. For the purpose of decoding (say) $g_{12} \equiv b_1 \oplus b_2$, we make use of the following $N-2$ weight-3 parity checks:

$$0 = (12) \oplus (23) \oplus (13) = (12) \oplus (24) \oplus (14) = \cdots = (12) \oplus (2N) \oplus (1N), \tag{2}$$

where we've used (ij) as a shorthand for g_{ij}. These checks provide us with $N-2$ independent ways to recover the logical value of $b_1 \oplus b_2$, namely

$$b_1 \oplus b_2 = (13) \oplus (23) = (14) \oplus (24) = \cdots = (1N) \oplus (2N). \tag{3}$$

(Of course, g'_{12} itself provides another independent way to recover $b_1 \oplus b_2$, but to keep our analysis simple we will not make use of g'_{12} here.) Since $g'_{ij} \neq g_{ij}$ with probability ε, each $g'_{1j} \oplus g'_{2j} \neq g_{ij}$ with probability

$$\varepsilon^* := 2\varepsilon(1-\varepsilon) \leq 1/2. \tag{4}$$

Therefore, g_{12} is protected by a length-$(N-2)$ classical repetition code, in which the $N-2$ bits flip independently with probability ε^*. This repetition code can be decoded through simple majority voting. The probability of a decoding error for g_{12} can be estimated from the Chernoff bound:

$$p_{\text{fail}} \leq \exp\left(-2(N-2)\left(\frac{1}{2}-\varepsilon^*\right)^2\right). \tag{5}$$

This is not the tightest possible Chernoff bound, and using additional information such as the observed value of g'_{12} will only improve the success probability. However, eq.(5) already illustrates our main point: the probability of a decoding error for any $b_i \oplus b_j$ decays exponentially with N. A simple union bound constrains the probability with which *any* of the $N-1$ bits are decoded incorrectly:

$$p_{\text{fail}}^{\text{total}} \leq (N-1)\exp\left(-2(N-2)\left(\frac{1}{2}-\varepsilon^*\right)^2\right). \tag{6}$$

Including g'_{12} in the decoding algorithm surely improves the accuracy of our estimate of $b_1 \oplus b_2$, and including higher-weight parity checks such as $0 = (12) \oplus (23) \oplus (34) \oplus (14)$ can yield further improvements. Following a pragmatic approach to using such information, we have implemented *belief propagation* (BP) [4], a fairly standard decoding heuristic for LDPC codes. BP efficiently approximates MLE decoding when the constraint graph is a tree, and sometimes works well in cases where the graph contains closed loops.

In BP, a marginal distribution is assigned to each variable, and updated during each iteration based on the values of neighboring variables. A consistent neighborhood reduces the entropy of the marginal whereas an inconsistent neighborhood may increase the entropy or even change a variable's most likely value. In our implementation, the initial configuration is the $\vec{g}'$ observed in a given run of the experiment, where each g'_{ij} is assumed to be correct with probability $1-\varepsilon$; two bits of $\vec{g}$ are considered to be neighbors if they share an index. A single iteration of belief propagation includes the majority vote on variable pairs $g'_{ik} \oplus g'_{jk}$ which we have already discussed, as well as the value g'_{ij} itself, to locally estimate the likelihood of each value for g_{ij}. To decode, beliefs are updated repeatedly until they converge to stable values. The probability of a decoding error is plotted in Fig. 1 as a function of ε and the number N of encoded spins. We include the Matlab code implementing the BP decoder and producing the benchmark figure as part of the arXiv source.

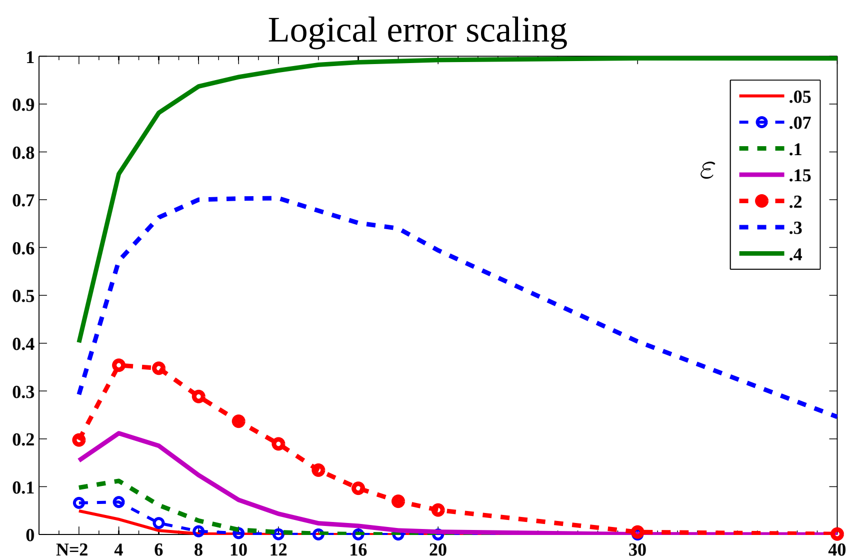

Figure 1: Performance of iterative BP decoding algorithm. The probability of a decoding error is plotted as a function of the number N of encoded spins, for various values of the physical error probability ε. Each data point was obtained by averaging over 5000 noise realizations, and for each realization the BP algorithm was iterated five times, incorporating information about loops up to length $33 = 2^5 + 1$. The decoding performance is significantly better than for a single BP iteration, where only loops of size ≤ 3 are considered. The logical error probability starts at $p_{\rm fail}^{\rm total} = \varepsilon$ for $N = 2$ and rises with N until the onset of exponential decay, which begins for a smaller value of N than indicated in eq.(6).

We conclude that the architecture proposed in [1], and the decoding method proposed here, provide good protection against i.i.d. noise in the readout of the physical spins, assuming an error probability ε for each physical spin which is independent of the total number N of encoded spins.

But is this characterization of the noise appropriate in this physical setting? Though quantum error-correcting codes might be invoked to improve accuracy [5, 6], no truly scalable scheme for quantum annealing has been proposed [7]. How well the Lechner *et al.* architecture performs under realistic laboratory conditions is a question best addressed by experiments.

We thank Wolfgang Lechner for useful comments and discussions. FP and JP gratefully acknowledge funding provided by the Institute for Quantum Information and Matter, a NSF Physics Frontiers Center with support of the Gordon and Betty Moore Foundation, and by the Army Research Office.

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